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Abstracts

Spectral Phases of the Erdös-Rényi graph

Johannes Alt
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We consider the Erdös-Rényi graph on N vertices with expected degree d for each vertex It is well known that the structure of this graph changes drastically when d is of order $\log N$. Below this threshold it develops inhomogeneities which lead to the emergence of localized eigenvectors, while the majority of eigenvectors remains delocalized.

In this talk, I will explain our results in both phases and present a phase diagram depicting them. I will also mention our results about the Schrödinger time evolution with the adjacency matrix as Hamiltonian. This is based on joint works with Raphael Ducatez and Antti Knowles.

The B2 index of galled trees

François Bienvenu
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In recent years, there has been an effort to extend the classical notion of phylogenetic balance, originally defined in the context of trees, to networks. One of the most natural ways to do this is with the so-called B2 index. In this talk, we study the B2 index for a prominent class of phylogenetic networks: galled trees. We show that the B2 index of a uniform leaf-labeled galled tree converges in distribution as the network becomes large and characterize the corresponding limiting distribution. This is the first time that a balance index has been studied to this level of detail for a random phylogenetic network. This work uses two different and independent approaches, each with its advantages: analytic combinatorics, and local limits. In this talk, we present the local limit approach.

Convergence of classical and multilevel variants of the Uzawa algorithm for general saddle point problems

Lori Badea
Simion Stoilow Institute of Mathematics of Romanian Academy, Romania

We analyze and compare the convergence of classical and multilevel variants of the Uzawa algorithm for general saddle point problems. The multilevel variant has similar iterations to the classical one, but the solution of the first equation is only approximated by inner iterations instead of an exact calculation. For both variants, classical and multilevel, explicit formulas for the convergence conditions and convergence rates are given.

It is proved that when the number of inner iterations of the multilevel Uzawa method tends to infinity, its convergence condition and convergence rate coincide with those of the classical Uzawa method. By writing the convergence rate of the multilevel method as a function of the number of inner iterations, we prove that it either has a minimum point or is an increasing function. Consequently, the multilevel method with very few inner iterations converges better than the classical method.

Numerical experiments carried out for the driven-cavity Stokes problem are in very good agreement with the theoretical results.

Oscillations in a delay differential equations model for blood disorders under periodic treatment

Irina Badralexi
University POLITEHNICA of Bucharest, Romania

We explore the presence of positive periodic solutions in a model that describes a myeloid cell line blood disorder. The dynamics of both healthy and malignant cells, along with the immune system's response and the effects of treatment, are modeled using a system of delay differential equations (DDEs). Using Krasnoselskii's approach, we provide parameter conditions that ensure the existence of positive periodic solutions.

Testing statistical hypotheses for Markov chains

Vlad Ştefan Barbu
University of Rouen, France

In this presentation we are concerned with a class of hypotheses tests for goodness of fit and homogeneity between general order Markov chains. These tests are based on the family of weighted Φ-divergences. A weight matrix treats the issue of the presence (or not) of prior information on the transitions of the system.

These measures allow us to focus on specific subsets of states (or of transitions) without, at the same time, loosing the other part of information. With this method we achieve a significantly more sensitive test than the classical ones, with comparable error rates. The appropriate asymptotic theory is presented according with Monte Carlo simulations for assessing the performance of the proposed test statistics.

Acknowledgement: this is a joint work with Thomas GKELSINIS (LMRS, University of Rouen - Normandy, France).

Interactions in the Lorentz force equation

Cristian Bereanu

University of Bucharest and Simion Stoilow Institute of Mathematics of the Romanian Academy, Romania

In this talk we consider the Lorentz force equation with a Kepler type electric potential and a smooth magnetic potential which are periodic in time If the electric and the magnetic potentials have "nice" interactions at infinity, then the Poincaré action functional has a sequence of periodic solutions with a given period. To prove our main result - using the Lusternik - Schnirelman category and Ekeland's variational principle - we develop a Lusternik - Schnirelman strategy for the Poincaré nonsmooth action functional associated to the Lorentz force equation.

Equilibrium conditions and existence of energy minimizers for higher order models of nonlinear elastic shells

Mircea Bîrsan

Octav Mayer Institute of Mathematics of the Romanian Academy, Romania and University of Duisburg-Essen, Germany

Within the nonlinear theory of elastic shells, we consider a recently introduced Cosserat shell model incorporating higher order effects Thus, we investigate 6-parameter models (allowing for transverse shear and in-plane drill) made of Cosserat materials Unlike the Kirchhoff-Love-Koiter models or the Reissner-Mindlin shells, the general 6-parameter shell model is mathematically well-posed.

Also, from an engineering point of view, 6-parameter shells have certain advantages: for instance, the imposition of boundary conditions is transparent, and these shells can easily be coupled with beam elements. This is due to the presence of the microrotation tensor field, which is an independent kinematical variable and allows for three additional degrees of freedom Hence, a numerical code implementing such models must be able to handle the rotation map.

We investigate several specific constitutive models for isotropic 6-parameter shells and show the coercivity of the areal strain energy density. Applying the direct methods of the calculus of variations we prove the existence of minimizers to the variational equilibrium problems. We also discuss the relevant Legendre-Hadamard conditions for 6-parameter shells as necessary conditions for energy minimizers.

These properties allow us to implement the Cosserat shell model using geometric finite elements, and to obtain numerical solutions to nonlinear shell problems with orientable or non-orientable midsurfaces.

Drug Resistance Pathways in Chronic Myeloid Leukemia

Ana-Maria Bordei

National Institute for Aerospace Research, Bucharest, Romania

We present a delay-differential equations model which describes the evolution of leukemic and healthy cells in chronic myeloid leukemia (CML) The system includes the action of the immune system and the influence of treatment with Imatinib. We model the situation in which the cells develop resistance to treatment by a Goldie-Coldman law.

Accelerated iterative algorithms for the Cauchy problem in steady-state anisotropic heat conduction

Mihai Bucataru
University of Bucharest, Romania

In the framework of stationary anisotropic heat conduction (the Laplace-Beltrami equation) without heat sources, we investigate both theoretically and numerically the acceleration of the two iterative algorithms of Kozlov et al for the accurate, convergent and stable reconstruction of the missing temperature and normal heat flux on an inaccessible boundary of the domain occupied by a solid from the knowledge of Cauchy data on the remaining and accessible boundary. The inverse Cauchy problem is reformulated into an equivalent fixed-point problem, involving analysis of convergence criteria and determination of optimal relaxation parameters within a suitable function space.

The numerical implementation is realised for two-dimensional homogeneous anisotropic solids using the finite element method and confirms a significant reduction in the number of iterations and hence CPU time required for the two relaxation algorithms proposed to achieve convergence, for both exact and perturbed Cauchy data. This is a joint work with Liviu Marin.

Flagfolds: multi-dimensional varifolds to handle discrete surfaces

Blanche Buet
Université Paris-Saclay, France

We propose a natural framework for the study of surfaces and their different discretizations based on varifolds Varifolds have been introduced by Almgren to carry out the study of minimal surfaces. Though mainly used in the context of rectifiable sets, they turn out to be well suited to the study of discrete type objects as well. While the structure of varifold is flexible enough to adapt to both regular and discrete objects, it allows to define variational notions of mean curvature and second fundamental form based on the divergence theorem.

Thanks to a regularization of these weak formulations, we propose a notion of discrete curvature (actually a family of discrete curvatures associated with a regularization scale) relying only on the varifold structure. We performed numerical computations of mean curvature and Gaussian curvature on 3D point clouds to illustrate this approach. Though flexible, varifolds require the knowledge of the dimension of the shape to be considered. By interpreting the product of the Principal Component Analysis, that is the covariance matrix, as a sequence of nested subspaces naturally coming with weights according to the level of approximation they provide, we are able to embed all d-dimensional Grassmannians into a stratified space of covariance matrices.

Building upon the proposed embedding of Grassmannians into the space of covariance matrices, we generalize the concept of varifolds to what we call flagfolds in order to model multi-dimensional shapes.

Fractional diffusion limit for a kinetic Fokker-Planck equation with diffusive boundary conditions in the half-line

Loïc Béthencourt
Université Côte d'Azur, France
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In this talk, we will investigate the long-time behavior of the critical kinetic Fokker-Planck equation. After reviewing the results for this equation in the whole plane, we will focus on the dimension $d = 1$ and the domain $\mathbb{R}_+$ with diffusive boundary conditions. From a probabilistic perspective, this amounts to study a random particle with position $(X_t)_{t\geq 0}$ living in $\mathbb{R}_+$, whose velocity $(V_t)_{t\geq 0}$ is a positive recurrent diffusion with heavy-tailed invariant distribution when the particle lives in $(0, \infty)$. When it hits the boundary $x = 0$, the particle restarts with a random strictly positive velocity.

Our main result states that the properly rescaled position process converges weakly to a stable process reflected on its infimum.

Uncertainty principles and sharp stability of functional inequalities

Cristian Cazacu
University of Bucharest and ISMMA, Romanian Academy

We present some useful functional inequalities in spectral theory of differential operators, in the study of partial differential equations or the stability of physical systems (motivated by quantum mechanics). We focus on two fundamental uncertainty principles (Heisenberg-Pauli-Weyl principle and Hydrogen principle). Mathematically, they are part of the wide class of inequalities of the Caffarelli-Kohn-Nirenberg type that we apply to either scalar or vector fields.

The best constants and extremizers are obtained In addition, we show sharp stability results in L^2 for such functional inequalities.

This exposure is based on joint works written in collaboration with Joshua Flynn (MIT, USA), Nguyen Lam (Memorial University of Newfoundland, Canada) and Guozhen Lu (University of Connecticut, USA).

On a fractional differential inclusion arising from HIV models

Aurelian Cernea
University of Bucharest, Romania

Mathematical models have been proven valuable in understanding the dynamics of HIV infection Perelson et al developed a simple model for the primary infection with HIV.

Since fractional-order models have been proved to be more accurate than integer models, i.e. there are more degrees of freedom on fractional order models, afterwards it was introduced a fractional order into a model of HIV infection. This model concerns Riemann-Liouville fractional derivative and boundary conditions in terms of Riemann-Stieltjes integral. Our aim is to study a set-valued version of this model and we obtain the existence of solutions in the case when the set-valued map has nonconvex values but it is assumed to be Lipschitz in the second and third variable.

Our result use Filippov's techniques; namely, the existence of solutions is obtained by starting from a given "quasi" solution. In addition, the result provides an estimate between the "quasi" solution and the solution obtained.

Zero Approximation of Hierarchical Models for Fluids in Pipes

Natalia Chinchaladze
Tbilisi State University, Georgia

We investigate dynamical problem of zero approximation of hierarchical models for fluids. Applying the Laplace transform technique, we reduce the dynamical problem to the elliptic problem which depends on a complex parameter τ and prove the corresponding uniqueness and existence results. Further, we establish uniform estimates for solutions and their partial derivatives with respect to the parameter τ at infinity and via the inverse Laplace transform show that the original dynamical problem is uniquely solvable.

Construction of Hunt processes by the Lyapunov method and applications to generalized Mehler semigroups

Iulian Cîmpean
University of Bucharest and Simion Stoilow Institute of Mathematics of the Romanian Academy, Romania

The aim of this talk is to reconsider the Lyapunov method for constructing Hunt processes on general spaces that are endowed with possible non-metrizable topologies, like the weak topology defined on an infinite dimensional Hilbert space. The main motivation is the study of generalized Mehler semigroups defined on a Hilbert space H, for which we prove that, under natural conditions, they correspond to Hunt processes on the same state space H, in contrast to previous results where an extension of the state spaces was required. As a concrete application, we consider a stochastic heat equation on $L^2(\mathbb{D})$ whose drift is given by the Dirichlet Laplacian on a bounded domain $\mathbb{D} \subset \mathbb{R}^d$, driven by a Levy noise whose characteristic exponent is not necessarily Sazonov continuous; in this case, we construct the corresponding Mehler semigroup and we show that it is the transition function of a Hunt process that lives on the original space $L^2(\mathbb{D})$ endowed with the norm topology.

In particular, we answer to the open problem of the existence of càdlàg Markov processes associated with generalized Mehler semigroup.

Neumann boundary controllability of a semilinear wave equation

Sue Claret

Université Clermont Auvergne, France

In this presentation, we consider the Neumann boundary controllability problem for a semilinear wave equation $\partial_{tt}y - \partial_{xx}y + f(y) = 0$. By a Schauder fixed-point argument, we give the existence of a semilinear control under an optimal growth assumption on f of the type $s \ln^2 s$. Then, assuming similar assumption on f', we construct a minimizing sequence which converges to a semilinear control.

Finally, we illustrate the results with some numerical experiments.

Steklov eigenvalue problems with weights for the *phi*-Laplacian

Nicuşor Costea

National University of Science and Technology POLITEHNICA Bucharest, Romania

In this talk we discuss the existence of nontrivial solutions for the problem

$$\operatorname{div}\left(\frac{\phi(|\nabla u|)}{|\nabla u|}\nabla u\right) = \frac{\phi(|u|)}{|u|}u$$

in a bounded smooth domain $\Omega \subset \mathbb{R}^N$, with a nonlinear boundary condition given by

$$\frac{\phi(|\nabla u|)}{|\nabla u|}\frac{\partial u}{\partial \nu} = \lambda V(x)\frac{\psi(|u|)}{|u|}u, \text{ on } \partial\Omega,$$

where $\phi, \psi : \mathbb{R} \to \mathbb{R}$ are odd strictly increasing homeomorphisms and V is a nonnegative weight function. The existence of a sequence of positive eigenvalues $\{\lambda_n\}$ such that $\lambda_n \nearrow \infty$ is established via the Ljusternik-Schnirelman Principle. A full description of the spectrum is provided when the function ψ has subcritical and either sublinear or superlinear growth w.r.t. ϕ.

Multi-dimensional coagulation models describing sedimentation

Iulia Cristian

University of Bonn, Germany

Coagulation equations describe the evolution in time of a system of particles that are characterized by their volume. Multi-dimensional coagulation equations have been used in recent years in order to include information about the system of particles which cannot be otherwise incorporated. Depending on the model, we can describe the evolution of the shape, chemical composition or position in space of clusters.

In this talk, we focus on a model that is inhomogeneous in space and contains a transport term in the spatial variable modeling the sedimentation of clusters. We prove local existence of mass-conserving solutions for a class of coagulation rates for which in the space homogeneous case instantaneous gelation (i.e., instantaneous loss of mass) occurs. This is based on a joint work with B. Niethammer (U Bonn) and J J. L Velázquez (U Bonn).

A glimpse at the marvelous world of Sobolev mappings into manifolds

Antoine Detaille
Lyon 1 University, France

In this talk, we will present Sobolev spaces of maps with values into manifolds, and explain some of the beautiful problems that arise in their study. Our starting point will be the following striking fact: unlike in the case of real-valued mappings, smooth maps with values into a compact manifold need not be dense in Sobolev spaces of maps with values into this manifold. The talk will contain very few proofs, but instead focus on the history of the problems under study, presenting the solution to some of them, which lead to beautiful statements, as well as some questions that have up to now firmly resisted to all attempts of resolution, providing challenging but simple-to-state open problems.

Localization for helical vortex filaments

Martin Donati
UMPA, ENS de Lyon et Institut Fourier, Université Grenoble Alpes, France

What happens in the wake of a wind turbine ? Due to its movement, the airflow after the turbine has a helical symmetry The dynamics of vortex filaments is the subject of many open mathematical problems, and the localisation property was only demonstrated in the case of vortex rings We prove the localisation property for helical vortex filaments, in the context of the 3D Euler equations in helical symmetry.

Spectral gap and embedded trees for the Laplacian of the Erdős-Rényi graph

Raphael Ducatez
University of Lyon, France

For the Erdös-Rényi graph of size N with mean degree $(1 + o(1)) \log N/(t + 1) \leq d \leq (1 - o(1)) \log N/t$ with t an integer, with high probability the smallest non-zero eigenvalue of the Laplacian is equal to $2 = 2\cos(\pi(2t + 1) - 1) + o(1)$. This eigenvalue arises from a small subgraph isomorphic to a line of size t linked to the giant connected component by only one edge. This is joint work with Renaud Rivier.

Community Detection and the (Hyper)Graph Stochastic Block Model

Ioana Dumitriu
University of California San Diego, USA

Community detection in complex networks is one of the fundamental problems in unsupervised learning, with applications from recommender systems to gene classification. In order to design state-of-the-art algorithms for community detection, as well as to predict and offer guarantees for their real-world performance, one often employs stochastic network models. The graph and hypergraph Stochastic Block Models (SBM) are a set of basic such models that have a lot of built-in flexibility and whose parameters can be adapted to suit various real-life networks (from social to biological and neural).

One crucial issue in the study of these models is represented by the "threshold" relationships among the defining parameters: these are sets of constraints (inequalities) which characterize the difficulty of the recovery problem and describe the best possible type of algorithmic performance on a network subject to these constraints. We will provide an overview of the problem, regimes of recovery, and existing literature for both the graph and hypergraph SBM, and then focus on exact and almost exact recovery for the hypergraph SBM. This represents joint work with Haixiao Wang and Yizhe Zhu.

Limitation of statistical tools in validation of random number generators

Simion Emil
National University of Science and Technology POLITEHNICA Bucharest, Romania

In cryptographic applications, the used cryptographic keys are generated through various processes: physical or algorithmic. From the perspective of cryptographic security, it is necessary to validate these technologies through statistical methods In this lecture, we will present a series of statistical tests that can support this process, as well as their limitations.

The case study is conducted with reference to the NIST SP 800-22 statistical test suite.

Some recent results on the discontinuous Galerkin approximation of Maxwell's equations

Alexandre Ern
ENPC, France

We present some recent results on the discontinuous Galerkin approximation of Maxwell's equations: an asymptotically-optimal error estimate for the problem posed in the frequency domain in second-order form, and the proof of spectral correctness for the eigenvalue problem in first-order form. Both problems hinge on a compactness property of the curl and divergence operators, and their numerical analysis crucially relies upon a duality argument originally proposed by Schatz in the context of the Helmholtz equation and conforming finite elements.

Modeling Immune Dynamics and Allergic Reactions in CML Therapy

Laurance Fakih
National University of Science and Technology POLITEHNICA Bucharest, Romania

This study presents a comprehensive mathematical model that delves into the complex dynamics of the immune system during Chronic Myeloid Leukemia (CML) therapy with imatinib. The model specifically focuses on elucidating the allergic reactions induced by imatinib, highlighting its impact on T helper (Th) cells and regulatory T cells (Tregs). By integrating cellular interactions, drug pharmacokinetics, and immune responses, this model aims to uncover the mechanisms behind the observed dominance of Th2 cells over Th1 and Treg cells, leading to allergic manifestations during treatment.

Employing a system of coupled delay differential equations, we explore the intricate interplay between healthy and leukemic cells, the modulation of T cell subsets by imatinib, and the subsequent emergence of allergic reactions. This approach allows for a detailed simulation of the temporal dynamics of cellular processes and drug interactions, providing a robust framework to study the progression of CML and the corresponding immune responses over time. Our model extends previous studies that primarily focused on the evolution of healthy and leukemic cells under treatment, by incorporating additional equations to account for allergic reactions.

This enhancement addresses a significant gap in the literature, offering deeper insights into the broader impact of imatinib on the immune system. Specifically, we investigate the hypothesis that the observed Th2 cell dominance contributes to the allergic side effects in patients undergoing imatinib therapy. The findings from this model not only advance our understanding of the immune dynamics during CML treatment but also have significant implications for optimizing therapeutic strategies.

By capturing the nuanced interactions between leukemic cells, immune cells, and imatinib, this study provides valuable insights into mitigating adverse immune-related side effects and improving patient outcomes. Through this mathematical framework, we aim to bridge the gap in knowledge regarding the immunological consequences of imatinib therapy, ultimately contributing to the development of more effective and safer treatment protocols for CML patients.

Facultative mutualisms and their resilience: global stability of coexistence equilibria

Paul Georgescu
Gheorghe Asachi Technical University of Iasi, Romania

We further pursue an investigation on an abstract model characterizing the dynamics of a general class of n-species facultative mutualisms that was initiated in P Georgescu, D. Maxin, H Zhang, *Threshold boundedness conditions for n-species mutualisms*, Nonlinearity 30 (2017) 4410–4427, establishing biologically relevant sufficient conditions for the global asymptotic stability of the coexistence equilibria. These conditions are given in terms of per-species limits of growth-to-loss ratios computed at higher population densities, complemented by either monotonicity or sublinearity inequalities, and are observed to hold for n-species versions of mutualistic models in current use. The specific modelling details that require either of these conditions being satisfied are outlined and discussed.

As mutualisms can enhance species diversification and facilitate stable coexistence via a plethora of mechanisms, our results are of potential interest to theoretical ecologists studying the coexistence of many interacting species and to conservationists aiming for rare species preservation

Numerical scheme for SPDEs with singular drift

Ludovic Goudenège
CNRS, France

I will present numerical schemes for stochastic partial differential equations with singular drift term having negative regularity, like Dirac distribution These results provide a speed of convergence for strong numerical approximation of such ill-posed SPDEs using a regularization effect from the space-time white noise.

A fresh look at algorithms for solving smooth multiobjective optimization

Sorin-Mihai Grad
ENSTA-Paris, France

We propose a new approach for constructing practical algorithms for solving smooth multi-objective optimization problems based on determining decreasing directions via suitable linear programming problems. The presented iterative method is specialized for unconstrained, sign constrained and linearly constrained multiobjective optimization problems. In all cases, the objective function values sequence is decreasing with respect to the corresponding nonnegative orthant, and every accumulation point of the sequence generated by the algorithm is a substationary point to the considered multiobjective optimization problem, becoming, under convexity assumptions, a weakly Pareto efficient solution.

Different to similar algorithms from the literature, the ones proposed in this work involve decreasing directions that are easily computable in polynomial time.

Numerical spectral analysis of Cauchy-type inverse problems: A probabilistic approach

Andreea Grecu
Gheorghe Mihoc-Caius Iacob Institute of Mathematical Statistics and Applied Mathematics, Romania

We introduce a probabilistic numerical approach for the reconstruction of the unknown boundary data of the steady state heat equation in a bounded domain in $\mathbb{R}^d$, having discrete measurements inside the domain and on a part of the boundary. We shall provide theoretical results which reveal that our approach is designed to spectrally approximate the inverse operator that we deal with. Finally, a parallel algorithm shall be presented together with numerical experiments.

This is based on joint work with Iulian Cîmpean and Liviu Marin.

Approximations of the best constants

Liviu Ignat
Simion Stoilow Institute of Mathematics of Romanian Academy, Romania

In this talk we consider some well known quotients related with either eigenvalue problems, Sobolev or Hardy's inequality. We consider the infimum of these quotients and their discrete analogous in a finite element subspace. We estimate the difference between the best constants above as the discretization parameter goes to zero.

Abstract parabolic equation in cylinders

Romain Joly
Université Grenoble Alpes, France

We consider a parabolic equation of the type $\partial_t u = (\partial_x^2 x - B)u$, where x belongs to the real line and where $B : D(B) \to Y$ is a sectorial operator defined in a Banach space. In the typical example, B is a Laplacian operator with respect to a transversal variable y belonging to a domain Ω, leading to a classical heat equation in a cylindrical geometry $R \times \Omega$. We study the Cauchy problem for the abstract PDE in the uniformly local Sobolev spaces.

This raises several questions of pure analysis, which will be considered in this talk from a non-specialist's point of view.

Riccati Pairs: An Alternative Approach to Hardy Inequalities

Sándor Kajántó
Babes-Bolyai University, Romania

We present the approach of Riccati pairs, a simple and elegant method for providing well-known and genuinely new Hardy inequalities on non-compact Riemannian manifolds. We show that the validity of these inequalities reduces to the solvability of corresponding Riccati-type differential inequalities. The proofs involve convexity arguments, divergence theorems, and comparison theorems.

Notably, they omit symmetrizations allowing for broad applicability: inequalities formulated on model space forms naturally extend to manifolds with smaller sectional curvature.

Non-Newtonian fluids and null controllability: the influence of finite stopping time

Geoffrey Lacour
University of Bordeaux, France

Non-Newtonian fluids are characterized by a viscosity that takes memory effects into account, meaning that the equations describing their flow are similar to Euler or Navier-Stokes equations with the addition of a nonlinear term. In many cases, the resulting equations are quasilinear, and the nonlinearity takes the form of p-growth.

In this talk, we will see that this viscosity term can imply stopping fluid flow in a finite time, and then we will look at the controllability of quasilinear parabolic equations with such nonlinearities. More specifically, we will show how the finite stopping time property can influence the null controllability of solutions.

Semiclassical Limit of the Bogoliubov-de Gennes Equations

Laurent Lafleche
ENS de Lyon, France

The Bogoliubov-de Gennes equations are a system of Schrödinger equations which are a generalization of the Hartree-Fock equation and appear for instance in the context of superconductivity. They take into account some purely quantum two-body pairing effect. In this talk, I will consider the time-dependent version of the Bogoliubov-de Gennes equation in the semiclassical approximation, that is when the Planck constant becomes negligible.

It leads to a two-particle kinetic transport equation coupled with a Vlasov equation. The proof uses the semiclassical optimal transport pseudo-metric introduced by Golse and Paul, and, in some regimes, gives a higher order two-body interaction term. This is joint work with Jacky J. Chong from Peking University and Chiara Saffirio from the University of Basel.

The Quantization Monte Carlo method for solving radiative transport equations

Laetitia Laguzet
CEA, France

We introduce the Quantization Monte Carlo method to solve thermal radiative transport equations with possibly several collision regimes, ranging from few collisions to massive number of collisions per time unit. For each particle in a given simulation cell, the proposed method advances the time by replacing many collisions with sampling directly from the escape distribution of the particle. In order to perform the sampling, for each triplet of parameters (opacity, remaining time, initial position in the cell) on a parameter grid, the escape distribution is precomputed offline and only the quantiles are retained.

The online computation samples only from this quantized (i.e., discrete) version by choosing a parameter triplet on the grid (close to actual particle's parameters) and returning at random one quantile from the precomputed set of quantiles for that parameter. We first check numerically that the escape laws depend smoothly on the parameters and then implement the procedure on a benchmark with good results.

Stability in shape optimization under convexity constraint

Jimmy Lamboley
Paris Sorbonne University, France

The notion of stability for functional and geometric inequalities has gained a lot of interest in recent years In the case of shape optimization (where one minimizes an energy whose variable is a subset of $\mathbb{R}^n$), the question of stability can be stated this way: if a domain is the unique minimizer of some energy, can we know that domains whose energy is close to the value of the minimum, are close to the unique minimizer? We would also like to quantify this. As an example we will show how this is done for the classical isoperimetric inequality, with a strategy that relies on computations of second order derivatives and regularity theory. We will then focus on how this strategy can apply to less standard energies when we restrict ourselves to convex shapes. This leads us to study the regularity theory of quasi-minimizers of the perimeter under convexity constraint. This is a joint work with Raphaël Prunier.

On global solutions for fluid dynamics SPDEs

Oana Lang
Babes-Bolyai University, Romania

A large class of partial differential equations exists in the literature for which global existence of solutions does not necessarily hold, with several such examples coming from geophysical fluid dynamics In this talk we will discuss conditions under which stochastic versions of the underlying equations can ensure existence of a global solution This is joint work with Dan Crisan and it is based on the paper available here: https://arxiv.org/abs/2403.05923.

Evolution time of stochastic systems with multiple final sequences of states

Alexandru Lazari
Moldova State University, Republic of Moldova

Let consider a discrete stochastic system L with finite set of states V. At every discrete moment of time t the state of the system is $v(t)$. The system L starts its evolution from each state v with given probability $p(v)$. Also, the transition matrix P is provided, such that the stochastic system L passes from one state u to another state v with given probability $P(u, v)$ and the transition time is unitary.

Additionally, r different sequences of states are given and the stochastic system L stops as soon as an arbitrary sequence from this list is reached. The system L represents a stochastic system with multiple final sequences of states and the time T, when the system L stops, represents its evolution time. It extends the zero-order Markov process with multiple final sequences of states, which was studied in [1] A. Lazari, *Zero-Order Markov Processes with Multiple Final Sequences of States*, Bul. Acad. Stiinte Repub. Mold Mat., 2023, No 2(102), 110-115. The purpose of this paper is to provide numerical methods for efficient computation of the main probabilistic characteristics of the evolution time T. It is proved that the distribution of the evolution time T represents a homogeneous linear recurrence.

This enables us to use the advanced properties of these recurrences and the algorithms proposed in [1].

Long time behavior of some self-similar fragmentation equation

Antoine Lejay
INRIA Nancy, France

We give some insights on some self-similar fragmentation equations. First, we consider the existence of a measure-valued solution by using the Mellin transform and the moments problems. Second, we consider the asymptotic behavior of the moments using some tools from complex analysis and Tauberian theorems.

From a joint work with G. Agazzotti and M Deaconu.

Luenberger observers for the linearized water-wave equation

Pierre Lissy
École des Ponts ParisTech, France

The aim of a Luenberger observer is to reconstruct asymptotically the state of a system, based on partial measurements. This is a crucial tool in control theory, notably in order to stabilize a system by state feedback. Here, we will explain the difficulties that appear if we want to implement a Luenberger observer for the full LWWE on the torus, when the measurement is done on a subinterval of the torus.

Then, we will give a remedy that enables to reconstruct the low frequencies with an explicit exponential decay rate. This is a work in progress with Lucas Perrin.

Asymptotic behaviour of a one-dimensional avalanche model through a non-conservative coagulation-fragmentation equation and associated stochastic processes

Oana Lupaşcu-Stamate
Gheorghe Mihoc-Caius Iacob Institute of Mathematical Statistics and Applied Mathematics, Romania

We study the asymptotic behaviour of an avalanche model through a discrete non-conservative binary coagulation-fragmentation equation. Our model is based on self-organized critical systems and in particular on a simple sand pile model introduced by Bressaud and Fournier. The key point of our approach is a new interpretation of the avalanche phenomena by handling stochastic differential equations (SDE) with jumps.

Our model is completed by numerical simulations. These results are obtained jointly with Madalina Deaconu (Nancy, France).

Spherical interpolation of scattered data using least squares thin-plate spline and inverse multiquadric functions

Andra Malina
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In T. Cătinaş, A. Malina, *Spherical interpolation of scattered data using least squares thin-plate spline and inverse multiquadric functions*, Numer. Algor (2024), we have constructed some smooth functions defined over a sphere that interpolate large sets of scattered data, using some modified Shepard methods, the least squares thin-plate spline, and the inverse multiquadric functions. This interpolation problem is important as it appears in solving some problems related to some physical phenomenons, such as temperature, rainfall, pressure, ozone, or gravitational forces, measured at various points on the surface of the Earth; they can also be applied in modeling closed surfaces in CAGD.

The Shepard method is one of the best-suited methods for approximating large sets of data. The radial basis functions are suitable tools for scattered data, and for data that varies rapidly over short distances. We restrict our attention to the case of (conditionally) strictly positive definite kernels, including a polynomial component, motivated by the fact that approximations of this kind offer real advantages.

We study some properties of the obtained Shepard-type operators and the corresponding approximation errors. The numerical tests considered on different types of data prove the efficiency of the methods. Moreover, a physical phenomenon is investigated, namely temperature prediction on the Earth's surface, and the results show that this method is a powerful instrument for solving various problems that model real-life phenomena.

Infinite-Bin Model and the Longest Increasing Path in an Erdös-Rényi random graph

Bastien Mallein
University of Toulouse, France

Consider an oriented version of the Erdös-Rényi random graph on the set of vertices $\{1, ...n\}$. For each pair $\{i, j\}$ with $i < j$, an edge from i to j is independently added to the graph with probability p. The length of the longest path in such a graph grows linearly with the number of vertices in the graph, and the growth rate is a deterministic function C of the probability p of presence of an edge. Using a coupling introduced by Foss and Konstantopoulos between this random graph and a particle system called the "infinite-bin model", we study some properties of the function C, including its analyticity on (0,1], its series expansion around 1, and its asympototic behaviour around 0.

Nonlinear fluids and Lipschitz truncations

Claudiu Mîndrilă
BCAM Bilbao, Spain

We provide existence of very weak solutions and a-priori estimates for steady flows of non-Newtonian fluids of Stokes and Navier-Stokes type when the right-hand sides are not in the natural existence class. This includes stress laws that depend non-linearly on the shear rate of the fluid like power-law fluids. To obtain the a-priori estimates we make use of a refined solenoidal Lipschitz truncation that preserves zero boundary values.

We provide also estimates in (Muckenhoupt) weighted spaces which permit us to regain a duality pairing, which than can be used to prove existence of solutions. Joint work with Sebastian Schwarzacher (Uppsala and Prague).

Discrete Laplacians on the hyperbolic space - a compared study

Dragoş Manea
Simion Stoilow Institute of Mathematics of the Romanian Academy, Romania

The main motivation behind this work stems from a notable gap in the existing literature: the absence of a discrete counterpart to the Laplace-Beltrami operator on Riemannian manifolds, which can be effectively used to solve PDEs. We consider that the natural approach to pioneer this field is to first explore one of the simplest non-trivial (i.e., non-Euclidean) scenario, specifically focusing on the 2-dimensional hyperbolic space $\mathbb{H}^2$. To this end, we present two variants of discrete finite-difference operator tailored to this constant negatively curved space, both serving as approximations to the (continuous) Laplace-Beltrami operator within the L^2 framework.

Moreover, we prove that the discrete heat equation associated to both aforesaid operators exhibits stability and converges towards the continuous heat-Beltrami Cauchy problem on $\mathbb{H}^2$. Eventually, we illustrate that a discrete Laplacian specifically designed for the geometry of the hyperbolic space yields a more precise approximation and offers advantages from both theoretical and computational perspectives.

Geometric rigidity of mappings and its applications in nonlinear elasticity

Cristinel Mardare
Jacques-Louis Lions Laboratory, Sorbonne University Paris, France

A classical theorem of Liouville about mappings from an Euclidean space into itself states that if such a mapping is sufficiently smooth and its gradient is a special orthogonal matrix at every point, then the mapping is necessarily affine. A quantitative version of this theorem, established in 2002 by Friesecke, James and Müller, states that the distance in the Sobolev space $W^{1,p}(\Omega)$, $1 < p < +\infty$, from any mapping $\Phi : \Omega \subset \mathbb{R}^n \to \mathbb{R}^n$ of class $W^{1,p}$ to the set of all affine mappings from Ω into $\mathbb{R}^n$ is controlled by the distance in the Lebesgue space L^p from its gradient field $\nabla\Phi : \Omega \to \mathbb{R}^{n\times n}$ to the set of all matrix fields from Ω into the set of all special orthogonal matrices of order n. We will discuss the applications of this result in nonlinear elasticity.

Source estimates with boundary observations for reaction-diffusion systems

Alexandra Melnig
Octav Mayer Institute of Mathematics of the Romanian Academy and Alexandru Ioan Cuza University, Romania

We consider systems of reaction-diffusion equations coupled in zero order terms, with general homogeneous boundary conditions in domains with a particular geometry (annular type domains). We establish Lipschitz stability estimates in L^2 norm for the source in terms of the solution and/or its normal derivative on a connected component of the boundary. The main tools include appropriate Carleman estimates in L^2 norms, with boundary observations, and positivity improving properties for the solutions to parabolic equations and systems.

A mixed finite elements approximation of inverse source problems for the wave equation

Sorin Micu
University of Craiova, Romania

We consider an inverse problem for the linear one-dimensional wave equation with variable coefficients consisting in determining an unknown source term from a boundary observation. A method to obtain approximations of this inverse problem using a space discretization based on a mixed finite element method is proposed and analyzed. Its stability and convergence relay on a new uniform boundary observability property with respect to the discretization parameter.

Combinatorial and probabilistic aspects of maps with tight boundaries

Grégory Miermont
ENS Lyon, France

Counting maps (or cellular embeddings of graphs in a two-dimensional surfaces) is an important problem with ramifications in algebraic and bijective combinatorics, probability and statistical physics. In this talk, I will discuss some recent results obtained with Jérémie Bouttier and Emmanuel Guitter concerning the enumeration of planar maps with tight boundaries, meaning that the contour of the boundaries have minimal length in their free homotopy class. In turn, these combinatorial results entail some interesting probabilistic properties of natural models of random surfaces, most of which are still to explore.

Maps and 2-dimensional random planar geometries

Grégory Miermont
ENS Lyon, France

A map is a topological surface obtained by identifying in pairs the edges of a finite collection of polygons. Viewing maps as abstract combinatorial objects, the countable collection of maps of a given topology can be seen as a set of discrete metrics defined on a fixed topological surface, for instance, by endowing the set of vertices of a map with the combinatorial graph distance. Therefore, considering models of random maps, for instance, a uniformly random triangulation of the sphere with a fixed number $2n$ of triangles, provides a natural setting for studying random metrics.

In this context, a question of interest concerns the asymptotic geometry of such a random object as n goes to infinity. In this talk, we will review some of the relevant combinatorial aspects of maps, and discuss how it allows to define limiting objects that are, in a sense, canonical models of random 2-dimensional geometries.

On a family of PDEs related to the principal frequency of the p-Laplacian and the p-torsion problem

Mihai Mihăilescu
University of Craiova and ISMMA, Romanian Academy, Romania

In this talk we discuss the asymptotic behavior of solutions for a family of partial differential equations, which is constructed as a convex combination between the p-torsion problem and the eigenvalue problem of the p-Laplacian, on an open and bounded subset Ω of the Euclidean space $\mathbb{R}^D$. We show that the family of solutions possesses at least a subsequence which converges uniformly over $\overline{\Omega}$ to a positive limit function which is bounded from below by the positive ground state of the Infinity Laplacian that has the maximum equal to the inradius of Ω and from above by the distance function to the boundary of Ω. Moreover, we can show that in some cases the entire family of solutions converges to the distance function to the boundary of Ω while in other cases the family of solutions can possess two subsequences converging to two different limits.

This is a joint work with Marian Bocea and Denisa Stancu-Dumitru.

Optimal approximation of unique continuation

Mihai Nechita
Babes-Bolyai University, Romania

We consider numerical approximations of ill-posed elliptic problems with conditional stability. The notion of optimal error estimates is defined including both convergence with respect to discretization and perturbations in data. The rate of convergence is determined by the conditional stability of the underlying continuous problem and the polynomial order of the approximation space.

A proof is given that no approximation can converge at a better rate than that given by the definition without increasing the sensitivity to perturbations, thus justifying the concept. A recently introduced class of primal-dual finite element methods with weakly consistent regularisation is recalled and the associated error estimates are shown to be optimal in the sense of this definition. Joint work with Erik Burman (University College London) and Lauri Oksanen (University of Helsinki).

Elliptic problems with 'log-normal' random coefficients

Victor Nistor
University of Lorraine, France

We consider a family of elliptic problems whose coefficients define random variables in a certain probability space. We prove uniform estimates for the solutions that are polynomial in the norms of the coefficients. In particular, if the coefficients are 'log-normal' in a suitable sense, we show that (the norm of) the solutions define an integrable random variable.

(My talk is based on joint works with C. Bacuta, M. Kohr, S. Labrunie, Hengguang Li, and H. Mohsen.)

Minimal control time for 1D linear hyperbolic systems

Guillaume Olive
Institute of Mathematics, Jagiellonian University, Poland

In this talk we will discuss recent results on the minimal control time for one-dimensional (1D) first-order linear hyperbolic systems. We will present several situations where we have a formula for this time which is explicit and easy to calculate with respect to the parameters of the system (speeds, internal and boundary coupling matrices). The proofs rely on several ingredients: a canonical LU-type decomposition for the boundary coupling matrix, the compactness-uniqueness method and the backstepping method. This presentation is based on several works in collaboration with Long Hu.

A dissipative nonlinear Schrödinger model for wave propagation in the marginal ice zone

Emilian Parau
University of East Anglia, UK

The evolution of unidirectional random waves in the marginal ice zone is modelled with a nonlinear Schrödinger equation, with a frequency dependent dissipative term consistent with recent field observations. The modulation instability is also investigated with this new model. It is observed that the frequency-dependent dissipation breaks the symmetry between the dominant left and right sideband.

Higher order models are discussed.

A local limit theorem for oscillating random walks on $\mathbb{Z}$

Marc Peigne
CNRS Tours, France

In this talk, we present some recent results on the recurrence properties of theoscillating random walk on $\mathbb{Z}$ and the asymptotic behavior of their return probabilities. The main tool is a renewal theorem for aperiodic sequences of operators due to S Gouezel and some variations of this theorem.

Identification of the parameters in a modified SIRD epidemic model using ensemble neural networks

Marian Petrica
University of Bucharest, Romania

In this talk, we present a parameter identification methodology of the SIRD model, an extension of the classical SIR model, which considers the deceased as a separate category. In addition, our model includes one parameter which is the ratio between the real total number of infected and the number of infected that were documented in the official statistics. Due to many factors, like governmental decisions, several variants circulating, opening and closing of schools, the typical assumption that the parameters of the model stay constant for long periods of time is not realistic.

Thus, our objective is to create a method which works for short periods of time. In this scope, we approach the estimation relying on the previous 7 days of data and then use the identified parameters to make predictions. To perform the estimation of the parameters, we propose the average of an ensemble of neural networks.

Each neural network is constructed based on a database built by solving the SIRD for 7 days, with random parameters. In this way, the networks learn the parameters from the solution of the SIRD model. Lastly, we validate our approach of parameter identification using real data of Covid19, from March 2020 until December 2021, for Romania, Hungary, The Czech Republic and Poland.

Therefore, we manage to obtain estimates of the parameters from real data, for the above period and then we endorse our results by making short-term predictions for different periods of time, from 10 up to 45 days, for the number of deaths. The results are backed by a theorem which guarantees that we can recover the parameters of the model from the reported data. We believe this methodology can be used as a general tool for dealing with short term predictions of infectious diseases or in other compartmental models.

Localization of energies in Navier-Stokes models with reaction terms

Radu Precup
Babes-Bolyai University, Romania

We analyze a general class of coupled systems of Navier-Stokes type equations with variable coefficients and non-homogeneous terms of reaction type in the incompressible case Existence of solutions satisfying the homogeneous Dirichlet condition in a bounded domain and localization results for the corresponding kinetic energy and enstrophy are obtained by using a variational approach and the fixed point index theory The presentation is based on a joint paper with Mirela Kohr.

Fractional diffusion for kinetic equations

Majorlane Puel
University of Cergy Paris, France

The study of kinetic equations involves a lot of variables and in particular, the numerical illustration is difficult to handel For that purpose, on some special range of parameters, we can derive some simpler models I Will explain a spectral method that lead to the different models depending on wich terme of the equation are the leading one.

Analyse spectrale des familles de Bloch isolés dans un champ magnétique regulier

Radu Purice
Simion Stoilow Institute of Mathematics of the Romanian Academy, Romania

Pour un hamiltonien donné comme opérateur pseudodifferentiel magnétique périodique ayant une famille finie de niveaux de Bloch isolés, meme en absence des lacunes spectrales la séparant du reste du spectre, on démontre l'existence d'un Hamiltonien effectif associée quand un champ magnétique regulier est present.

Modelling HIV therapy and allergic reactions

Rodica Rădulescu
National University of Science and Technology POLITEHNICA Bucharest, Romania

Using delay differential equations, we develop a new model that describes the evolution of cells of the immune system's cells involved in allergies, the evolution of HIV viruses, of that of the infected and uninfected CD4+ cells and of cytotoxic T-lymphocytes during specific anti-retroviral therapy. Qualitative properties of the solutions, as essential non-negativity and global existence are proved A partial stability is provided.

The evolution of the infected cells and their relation to immune system during HIV treatment with a glimpse to avoid allergic reactions induced by the medication, are also assessed via numerical simulations.

Derivation and analysis of singular Vlasov equations

Frederic Rousset

Paris-Saclay University, France

We shall introduce a class of singular nonlinear Vlasov type equations where the regularity of the force field is at the level of the gradient of some hydrodynamics moments. In the first part of the talk we will show how these models can be formally obtained from more standard models of Mathematical Physics in particular the quasi-neutral regime for plasmas and the semiclassical regime of Hartree equations. In the second part of the talk we will discuss the well-posedness of these models and their rigorous derivation.

Propagation of smallness and spectral estimates on non-compact manifolds

Marc Rouveyrol

Paris-Saclay University, France

Spectral estimates consist in bounding the norm of frequency-localized functions by their norm on a smaller, so-called "sensor" set, up to some factor depending on the frequency threshold. They were first studied by Logvinenko and Sereda in the 1970s for flat Laplacians, in connection with the uncertainty principle. Recent investigations have focused on the manifold case, with tools drawing from spectral theory, harmonic analysis, geometric analysis and control theory for the heat equation. Beyond controllability results, applications of these estimates include spectral geometry and the study of random Schrödinger operators.

The aim of the talk will be to give an introduction to spectral estimates and their link with controllability, as well as to present the first high-frequency results on non-compact, non-flat Riemannian manifolds. In the model case of the hyperbolic half-plane, we will give an optimal equidistribution condition on the sensor set, equivalent to spectral estimates.

If time allows, we will discuss generalizations to hyperbolic surfaces, on which decay in the volume of balls creates additional difficulties Partly joint work with Alix Deleporte.

Resonances as a computational tool

Katharina Schratz

University Paris Sorbonne, France

A large toolbox of numerical schemes for dispersive equations has been established, based on different discretization techniques such as discretizing the variation-of-constants formula (e.g., exponential integrators) or splitting the full equation into a series of simpler subproblems (e.g., splitting methods). In many situations these classical schemes allow a precise and efficient approximation. This, however, drastically changes whenever non-smooth phenomena enter the scene such as for problems at low regularity and high oscillations.

Classical schemes fail to capture the oscillatory nature of the solution, and this may lead to severe instabilities and loss of convergence. In this talk I present a new class of resonance based schemes. The key idea in the construction of the new schemes is to tackle and deeply embed the underlying nonlinear structure of resonances into the numerical discretization.

As in the continuous case, these terms are central to structure preservation and offer the new schemes strong geometric properties at low regularity.

Rank revealing algorithms for general rational matrices: class of solutions and applications

Radu Ştefan
National University of Science and Technology POLITEHNICA Bucharest, Romania

A leading idea of the paper concerns the extension of the well-known rank revealing factorization of a constant matrix to rational matrix functions. The results are completely general and apply in particular to matrices which are polynomial, strictly proper or improper, rank deficient, with arbitrary poles and zeros including at infinity, having further impact on the practical implementation of modern control methods to various complex systems.

Regularity for a perfect elastic beam interacting with the Navier-Stokes equations

Pei Su
Paris-Saclay University, France

We are interested in the interaction of a viscous incompressible fluid with an elastic structure, where the structure is located on a part of the fluid boundary. It reacts to the surface forces induced by the fluid and deforms the reference domain to the moving domain. The fluid equations are coupled with the structure via the kinematic condition and the action-reaction principle on the interface. We consider a 1D perfectly elastic plate, deforming vertically in flat case, interacts with 2D Navier-Stokes equations, which thereby gives a hyperbolic evolution. We show the new regularity result for this parabolic-hyperbolic coupled system. It turns out that the "parabolic effect" of the fluid suffices to regularize the solution to the coupled fluid-structure system which is previously known for the Navier Stokes equations in fixed domains. This is a joint work with S. Schwarzacher (Uppsala University).

Iterative schemes for coupled flow and transport problems in porous media - Convergence and truncation errors

Nicolae Suciu
Tiberiu Popoviciu Institute of Numerical Analysis of the Romanian Academy, Romania

Nonlinearities of coupled flow and transport problems for variably saturated porous media are solved with explicit iterative L-schemes. Their behavior is analyzed with the aid of the computational orders of convergence. This approach allows highlighting the influence of the truncation errors in the numerical schemes on the convergence of the iterations.

Further, by using manufactured exact solutions, error-based orders of convergence of the iterative schemes are assessed and the convergence of the numerical solutions is demonstrated numerically through grid-convergence tests.

Nonlinear Dirichlet problem of non-local branching processes

Alexandra Teodor
National University of Science and Technology POLITEHNICA Bucharest, Romania

We present a method of solving a nonlinear Dirichlet problem with discontinuous boundary data and we give a probabilistic representation of the solution using the non-local branching process associated with the nonlinear term of the operator. Instead of the pointwise convergence of the solution to the given boundary data we use the controlled convergence which allows to have discontinuities at the boundary. The talk is based on a joint work with Lucian Beznea and Oana Lupaşcu-Stamate (Bucharest).

Inference for the nonlinear stochastic heat equation

Ciprian Tudor
Lille University, France

We consider the nonlinear stochastic heat equation with fractional Laplacian, driven by the Gaussian space-time white noise and we analyse the asymptotic behavior of the quadratic and higher order variations of its mild solution. The idea is to approximate the increments of the solution to the nonlinear heat equation with those of the solution to the linear heat equation (which is related to the fractional Brownian motion). Based on these variations, we construct estimators for several parameters that may appear in such a model: the drift and diffusion parameters or the parameter associated with the order of the fractional Laplacian.

Stein's method and the spatial average of the solutions to SPDE

Ciprian Tudor
Lille University, France

We present a variant of the Stein-Malliavin calculus that allows to estimate the asymptotic independence between sequences of random variables. We apply the method to analyze the asymptotic behavior of the solution to certain SPDEs, including the stochastic heat and wave equations.

Numerical Simulation of Fluid-Structure Interaction in the Left Heart with Reduced Valve Modeling

Marina Vidraşcu
INRIA, France

The four heart's chambers (left and right ventricle; left and right atria) manage the heartbeat and blood flow through the body. The chambers work with each other and other parts of the heart like valves and arteries to keep the heart pumping Simulate the fluid-structure interaction (FSI) in the heart is a challenging multiphysics and multiscale problem.

To address its computational complexity, various authors have proposed lcomputationally simplified approaches to partially solve this problem These approaches encompass complete electromechanical simulations of the heart using a 0D fluid [1, 2], kinematic uncoupling methods that impose displacements from electromechanical simulations onto the boundaries of the fluid cavities [3], and FSI approaches that consider either 0D [4] or 3D [5] models for the heart valves. However, many of these approaches have shortcomings, such as difficulties in properly capturing isovolumetric phases with less expensive numerical methods, and issues in handling enclosed domains for more costly numerical methods.

In this work, we propose to study the fluid-structure interaction (FSI) of the left heart, utilizing a mathematically consistent and novel reduced valve model. Additionally, consider the effect of the right ventricle by incorporating a 0D fluid representation in that cavity. We introduce a novel fluid-structure interaction coupling based on a Robin-Robin loosely coupled scheme [6].

Our investigation aims to explore the advantages of employing this method in accurately capturing isovolumic phases and achieving a reduced computational cost Ongoing work will incorporate the atria in the model 1.

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Recent results for the study of quasi-stationary distributions

Denis Villemonais
University of Lorraine, France

After an introduction to the notion of quasi-stationary distributions, I will present three different approaches for its study in a general context. The first approach is inspired by Meyn and Tweedie's work on conservative Markov processes, the second relies on spectral theoretical tools and the third one considers the problem of convergence in Wasserstein distance. Applications to perturbed dynamical systems, diffusion processes and measure valued Pólya processes will be presented.